Center for Mathematical Sciences

Technion - Israel Institute of Technology

THE MALLAT FAMILY FUND FOR RESEARCH IN MATHEMATICS

invites you to a

SPECIAL LECTURE SERIES

to be presented by

Professor William Arveson

University of California, Berkeley

The lectures will be held in

Room 232
Amado Mathematics Building
Technion - Israel Institute of Technology
Haifa, Israel

Lecture I: Monday 3 April, 2006 at 15:30

Operator theory and the K-homology of algebraic varieties

Let *X,Y,Z* be three mutually commuting operators acting on a common Hilbert space that satisfy a nonlinear equation of the form

(1) $$X^n + Y^n = Z^n,$$

for some *n* =2 ,3 ,....The *C* *-algebra generated by *X,Y,Z* is typically noncommutative,and can be viewed as a non-classical counterpart of the curve $V \subseteq \mathbb{C}^3$ defined by $x^n + y^n = z^n$.

Similarly, there are natural non-classical counterparts of more general algebraic varieties $V \subseteq \mathbb{C}^d$.

Starting from first principles, we describe a natural construction of "universal" operator solutions of equations like (1)and we describe the general properties of these operator solutions, focusing on the question: When does an operator solution of a system of equations like (1)determine an element of the *K* -homology of the associated classical variety *V*? We formulate this question as a concrete conjecture about self- commutators –such as *X* *X -XX *,X *Y -YX *,...,in example (1) –and describe recent progress on proving the conjecture in general.

Lecture II: Wednesday, 5 April, 2006 at 15:30

Standard Hilbert modules and linearization

A *standard Hilbert module* (over the polynomial algebra $A = \mathbb{C}[z_1, \ldots, z_d])$ is a Hilbert *A* -module obtained by first completing *A* relative to an inner product with appropriate properties, and then increasing the multiplicity of that completion. Standard Hilbert modules are very general –encompassing most historical examples such as the Hardy and Bergman modules of domains as well as non-subnormal modules associated with reproducing kernels. On the other hand, they also appear to provide an appropriate context for confronting many of the open problems of operator theory. In this talk we describe the basic properties of standard Hilbert modules and their quotient modules, focusing on the problem of establishing a Fredholm/index theory that is effective for the most basic examples that are associated with algebraic equations. The emphasis will be on concrete problems and conjectures.

Lecture III: Thursday, 6 April, 2006 at 15:30

Recent progress in noncommutative dynamics

In quantum theory, the flow of time or the group of motions with specified momentum are modeled by a one-parameter group of automorphisms of the algebra *B* (*H*)of all bounded operators on a Hilbert space *H* .Such automorphism groups are completely understood; they are classified by classical results of Stone, Wigner, and the multiplicity theory of Hahn and Hellinger. On the other hand, if one adds a natural notion of causality to such a flow on *B* (*H*),entirely new phenomena emerge. We have not yet seen all the possibilities and we do not know how to classify the possibilities we have seen. We describe how causal flows give rise to pairs of

E_0–semigroups and summarize what is known about the dynamics of E_0–semigroups, emphasizing connections with noncommutative probability theory and unsolved problems.